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# NUMERICAL MODELING IN CONJUGATE FORMULATION OF THE TEMPERATURE STATE OF THE LEADING EDGE OF HYPERSONIC FLYING VEHICLES UNDER DIFFERENT METHODS OF ORGANIZATION OF THE COOLING SYSTEM

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*Heat transfer in the region of the leading edge of a hypersonic flying vehicle has been calculated by finite-difference integration of the Reynolds equations closed by the low-Reynolds version of the k–ε model of turbulence. Integration has been performed using the explicit–implicit McCormack scheme. It is shown that blowing of a cooling agent through a porous insert can lead to a substantial decrease of heat flux. The Darcy equation was used in solution of the conjugate problem.*

Progress in air transport is impossible without further improvement of the flying vehicles employed in civil aviation. One of most promising areas is the project associated with designing of a hypersonic transport airplane [1]. By NASA estimates, this airplane, which is capable of transportation of 300 passengers to a distance of 10,000 km at a hypersonic velocity (M = 6–10), could be of interest for airlines, since it could become an adequate rival to supersonic airplanes of the Concord or Tu-144 types. It could be equipped by a hypersonic hydrogen ramjet engine. By its aerodynamic arrangement, a hypersonic flying vehicle (HFV) will greatly differ from orbital aircrafts (Buran, Space Shuttle), thus approaching the design of supersonic airplanes. At a cruising speed, an HFV must have an aerodynamic efficiency not less than three, which can be obtained by the use of small relative thicknesses of the wing and fuselage and extremely small radii of curvature of their leading edges (≈10 mm). Moreover, the lower surface of the body must play the role of the upper surface of a plane nozzle and the nose shock wave must be "attached" to the leading edge of the air intake of the engine. All these configurational solutions impose strict requirements upon thermal protection compared to both supersonic and orbital airplanes. In contrast to military rocket technology, thermal protection of HFVs must satisfy the following requirements: repeated use (elimination of replacement during service) of the system of thermal protection; erosion resistance to rain, snow, and dust; repairability under airfield conditions; high reliability; low cost of design and manufacture which does not reduce the HFV competitiveness.

The most serious considerations must be given to thermal protection of the most thermally stressed elements of the HFV structure: the fuselage nose, leading edges of a wing, tail fin, air intake of the engine, places of incidence of shock waves. Use of both passive thermal protection (thermoinsulating, radiating, and partially evaporating coatings) and semi-active (pumpless circulation of a cooling agent) can turn out to be inapplicable under these conditions. To reliably protect the zones of extremum heating it is suggested to use active methods in which the cooling agent is not only forced to the heating surface but also penetrates through the shell into the boundary layer of the incoming air flow.

**Mathematical Model.** The Reynolds number for an HFV flying at a height of 30,000 m with Mach number M = 8, which is determined by the radius of leading-edge rounding, has an order of $10^5$. Thus, for calculation of a flow near the leading edge of an HFV the model based on numerical integration of the Reynolds equations closed by one model of turbulence or another must be used. The fact that injection of the cooling agent in the region of the leading edge will inevitably facilitate flow turbulization speaks in favor of this use.

In the present paper, we used the conservative form of presentation of two-dimensional equations of gas dynamics in an arbitrary curvilinear system of coordinates. The low-Reynolds version of the $k$–ε model of turbulence

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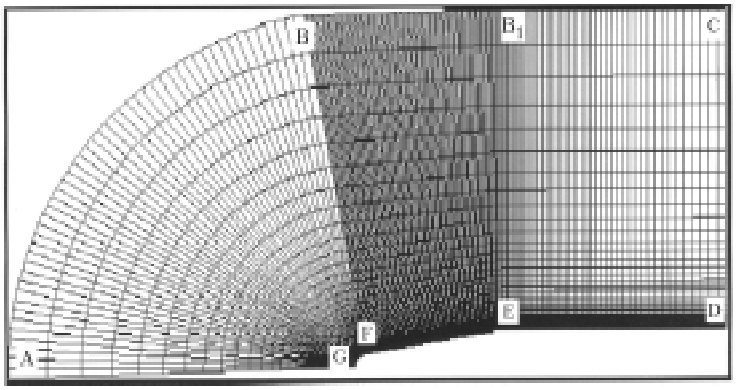

Fig. 1. Computation grid.

with corrections to the curvature of stream lines was used for determining the characteristics of turbulence [2]. To perform finite-difference integration of the initial system of equations we applied the two-step explicit–implicit MacCormack scheme of the second order of accuracy on both space and time coordinates [3].

In studying the thermal state of the permeable edge of an HFV, the discrete initial equations were solved in the computation domain presented in Fig. 1. On assignment of the initial conditions it was assumed that the flow inside the domain is uniform. In this case, the parameters of the gas at all computation nodes were taken to be equal to the parameters of the gas in a nondisturbed flow. At the inlet to the domain (section AB), the parameters of the flow were kept constant. "Soft" boundary conditions, i.e., the conditions of solution continuation, were set on the upper (BC) and outlet (CD) boundaries. On the lower boundary (AG), the condition of flow symmetry was set. In the study conducted, the computation domain was divided by the grid with thickening of nodes at the wall. The computation grid was a combination of the polar grid (sector ABFG), the grid generated by numerical solution of the Poisson equations ($FBB_1E$), and the Cartesian grid ($EB_1CD$); the grid contained $202 \times 61$ computation nodes. The minimum pitch near the wall was taken to be of an order of $10^{-4}$. It is noteworthy that the velocity, temperature, and density of the incoming flow were taken as the scales of nondimensionalization of the parameters. The half-thickness of the streamlined body was taken as a unit of linear size (ordinate of the point D) (Fig. 1).

**Modeling of the Thermal State of an Impermeable Thermally Insulated Edge.** To substantiate the necessity of using the system of cooling, we model a flow past the leading edge of an HFV flying at a height of 30,000 m with Mach number M = 8. We introduce the adiabaticity condition $(dT/dn)_w = 0$ as the boundary condition for the temperature on the solid surface. The distribution of the gasdynamic parameters along the generatrix of the edge is shown in Fig. 2.

Figure 2c gives the temperature distribution of the thermally insulated edge of the HFV. It is seen from the graph presented that on the entire surface under consideration the temperature exceeds the ultimately admissible temperature (1500 K). Thus, one system of thermal protection or another must be used for hypersonic flight to be executed.

To verify the correctness of the calculations made (within the framework of the adopted assumptions), we compare the calculated parameters of complete retardation of flow with their values obtained according to the theory of one-dimensional flow of an ideal gas. From the relations for the normal shock wave in the flow of gas with constant heat capacities we have

$$\frac{p_{02}}{p_1} = ((1+\delta)\,\mathrm{M}_1^2 - \delta)^{(\delta-1)/2\delta}\,\mathrm{M}_1^{(1+\delta)/\delta}\,(1-\delta)^{(1+\delta)/2\delta},$$

where $\delta = (\kappa - 1)/(\kappa + 1)$, $\kappa = 1.4$.

The temperature of gas retardation behind the shock wave does not change and remains equal to

$$T_{02} = T_{01} = T_1\left(1 + \frac{\kappa - 1}{2}\,\mathrm{M}_1^2\right).$$

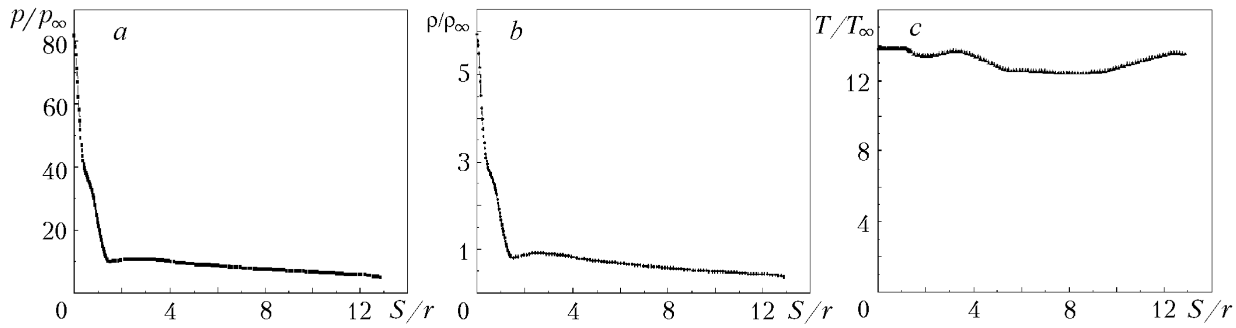


Fig. 2. Distribution of relative pressure (a), density (b), and temperature (c) along the generatrix of the edge.

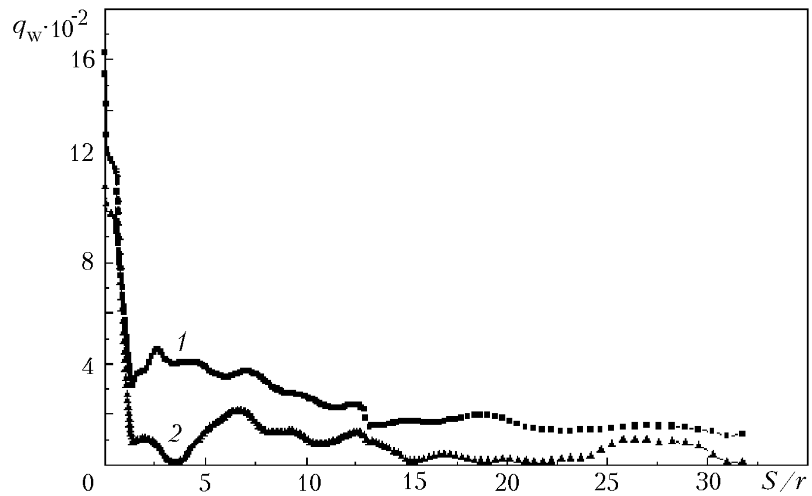


Fig. 3. Distribution of heat flux along the generatrix of the edge: 1) impermeable isothermal surface, $T$ = 1500 K; 2) permeable isothermal surface in equally distributed homogeneous injection at a rate $\bar{j}$ = 0.01.

At the Mach number M = 8: $p_{02}/p_1$ = 82.865 and $T_{02}/T_1$ = 13.8.

Moreover, it is known that with an unlimited increase of the Mach number the ratio of densities in the air medium behind the normal shock wave and in front of it will tend to six.

As is seen from Fig. 2, the flow parameters at the point of retardation, which are obtained from numerical modeling, correspond to their theoretical values.

**Modeling of the Thermal State of the Edge in Organization of Uniform Injection.** To estimate the value of the heat flux coming to the cooled leading edge of an HFV, we calculate the thermal state of the isothermal surface. In this case, the boundary condition for the wall temperature is $T_w$ = 1500 K. As is seen from Fig. 3 (curve 1), the heat flux has a considerable value, which near the forward retardation point exceeds 1600 kW/m$^2$.

To remove the thermal load, we inject the cooling agent into the rounded leading edge (GF) and the side surface. We make calculations for the relative mass velocity of injection (injection rate) $j = (\rho u)_w/(\rho u)_\infty$ = 0.01. We assume that uniform supply of the cooling agent is organized over the entire cooled surface. We also assume that the temperature of the cooling agent on the outer surface of the porous insert is equal to the temperature of the wall and amounts to 1500 K. The heat-flux distribution in flow past an impermeable isothermal surface and in uniformly distributed homogeneous injection is shown in Fig. 3 (curve 2).

It is seen from the graphs that the front part of the rounded edge is the most thermally stressed. Due to the organization of injection of the cooling agent through the porous insert, we succeeded in decreasing the heat flux at the forward stagnation point by about 2.5 times.

**Modeling of the Thermal State of the Edge in Organization of Injection with Account for the Characteristics of the Porous Material, the Wall Thickness, and the Pressure of the Cooling Agent in the Supply Line.** The gas motion in a porous medium is described by the Darcy equation [4]:

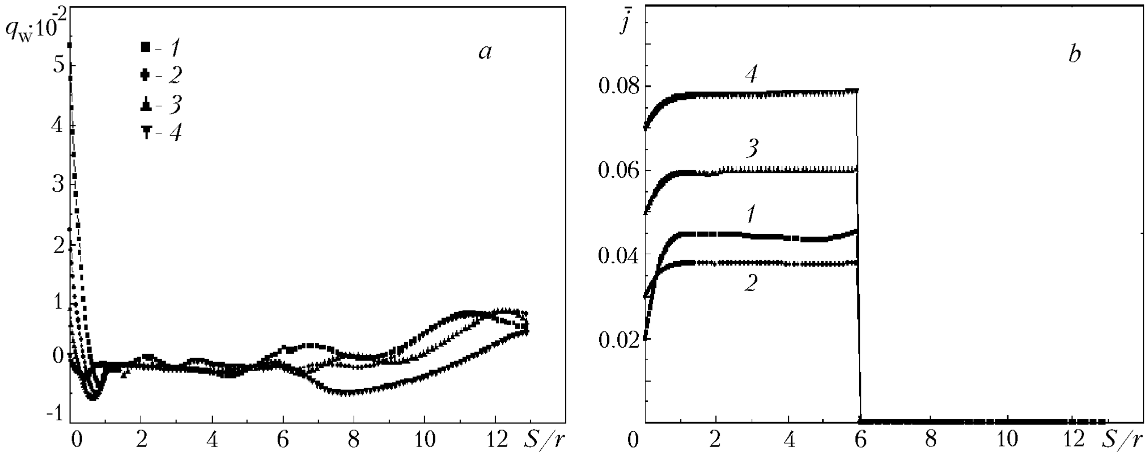


Fig. 4. Distribution of heat flux (a) and relative mass velocity of injection (b) along the generatrix of the edge: 1) $\bar{j}_0$ = 0.02 and $\Delta$ = 1 mm; 2) 0.03 and 3; 3) 0.05 and 3; 4) 0.07 and 3.

$$-\frac{dp}{dZ} = \alpha\mu u + \beta\rho u^2 , \quad \mu = \mu\,(T, p) .$$

For the case of a compressible ideal gas flow, for which the equation of state $p = \rho RT$ holds, the Darcy equation can be presented in a more convenient form:

$$-\frac{d\,(p^2)}{dZ} = 2\alpha\mu GRT + 2\beta G^2 RT .$$

We assume that the temperature of the cooling agent is equal to the temperature of the porous wall ($T$ = 1500 K), whose thickness is $\Delta$ = 1–3 mm. In this case, for the ratio of pressures at the inlet to and outlet from the porous element we can write

$$p_1^2 - p_2^2 = 2\Delta RT\,(\alpha\mu G + \beta G^2) .$$

We determine the pressure at the inlet $p_1$ which is necessary for creating a relative mass velocity of injection $\bar{j}$ = 0.01 at the point of flow retardation.

Let the cermet material have the following characteristics:

$$\alpha = 2.4\cdot 10^8\cdot \Pi^{-5.6}\ \mathrm{m}^{-2} , \quad \beta = 2.7\cdot 10^3\cdot \Pi^{-5.6}\ \mathrm{m}^{-1} , \quad \Pi = 0.3 .$$

For a 1-mm-thick porous wall made of this material the necessary difference of pressures is $\Delta p \approx 23{,}843$ Pa and the coolant pressure (with no regard for pressure variation at the forward stagnation point due to injection) $p_1$ = $1.23034\cdot 10^5$ Pa.

The computational algorithm which realizes the combined solution of the Reynolds equations and the Darcy equation is constructed as follows. After each iteration (time step) of the finite-difference procedure of the solution of the Reynolds equations we find the pressure distribution at the outlet from the porous medium. Its value is substituted into the Darcy equation. Then, from the latter we find the momentum distribution along the wall. Then the boundary conditions for the density and the velocity components in the Reynolds equations are corrected and the calculation goes on.

Use of this system of cooling makes it possible to virtually completely remove the heat flux on the side surface and to considerably (≈2.5 times) decrease it at the forward point. Here, the rate of injection is 0.01 and that on the side surface is ≈0.022. To decrease thermal loads at the forward point of the fuselage the intensity of injection must be increased. In this case, for the rate of the cooling agent to be decreased, the length of the porous insert can be limited to a value of $S/r$ = 6 and on the remaining surface a passive system of heat protection can be used. To produce injection, which at the critical point of flow is 0.02, we need a pressure difference $\Delta p \approx 46{,}458.26$ Pa. Then the pressure of the cooling agent must be equal to $p_1 \approx 1.4565 \cdot 10^5$ Pa. In this case, it is possible to decrease the heat flux at the forward stagnation point by about 3.5 times.

We determine the value of the injection rate necessary for complete removal of heat flux at the stagnation point. Figure 4 gives the characteristics of heat and mass transfer near the leading edge of the HFV at different pressures of the cooling agent and a thickness of the porous insert of $\Delta$ = 1–3 mm.

The necessary pressure difference in the cases under consideration is:

at $\bar{j}_0 = 0.03$ $\quad \Delta p \approx 1.548 \cdot 10^5$ Pa $\quad \bar{q}_0 \approx 0.12$;
at $\bar{j}_0 = 0.05$ $\quad \Delta p \approx 2.365 \cdot 10^5$ Pa $\quad \bar{q}_0 \approx 0.05$;
at $\bar{j}_0 = 0.07$ $\quad \Delta p \approx 3.133 \cdot 10^5$ Pa $\quad \bar{q}_0 \approx 0.07$.

On the basis of the calculations made we can draw the following main conclusions:

1. Porous cooling is an effective means of decreasing thermal loads caused by aerodynamic heating of hypersonic flying vehicles.

2. To remove thermal loads from the side surface of a 10% wedge at a flight Mach number M = 8 at a height of 30,000 m it is necessary to provide injection of the cooling agent at a rate $\bar{j}$ = 0.01–0.02.

3. For complete removal of thermal loads at the forward stagnation point it is necessary to provide injection at a rate $\bar{j} \geq 0.07$.

4. At the given value of pressure of the cooling agent in the supply line the distribution of the injection rate along the surface of the porous insert will greatly depend on the pressure distribution on the outer surface and also on the characteristics of the porous material (viscous or inertia coefficients of resistance).

5. For rational consumption of the cooling agent it is necessary to organize differentiated injection of the cooling agent with the highest value of $\bar{j}$ at the stagnation point of the flow.

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## NOTATION

$k$, kinetic energy of turbulent fluctuations, m$^2$/sec$^2$; $\varepsilon$, rate of turbulent energy dissipation, m$^2$/sec$^3$; $p$, pressure, Pa; $T$, temperature, K; $n$, distance along the normal to the wall, m; $R$, gas constant, J/(kg·K); $p_1$ and $p_2$, gas pressure in front of and behind the shock wave, Pa; $p_{01}$ and $p_{02}$, pressure of the retarded gas in front of and behind the shock wave, Pa; $\rho$, density, kg/m$^3$; $u$, velocity, m/sec; $S$, length of the generatrix, m; $r$, radius of the nose rounding, m; Re, Reynolds number; M, Mach number; $M_1$, Mach number in front of the shock wave; $\kappa$, adiabatic index for air; $q$, heat-flux density, kW/m$^2$; $\bar{q}_0$, ratio of the heat flux at the stagnation point to the flux in the absence of injection; $\bar{j}$, relative mass velocity of injection (injection rate); $\bar{j}_0$, injection rate at the forward stagnation point; $X$, $Y$, dimensionless coordinates; $Z$, coordinate, m; $\mu$, dynamic coefficient of viscosity, (N·sec)/m$^2$; $\alpha$ and $\beta$, viscosity and inertia coefficients of resistance of the porous material, m$^{-2}$ and m$^{-1}$; $G$, specific mass flow rate of gas, kg/(m$^2$·sec); $\Pi$, porosity of the material. Indices: w, wall; 1 and 2, parameters in front of and behind the shock wave; 0, parameters of the retarded flow; $\infty$, parameters of the nondisturbed flow.

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